Massive gravitational waves from the R^2 theory of gravity: production and response of interferometers

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February 2, 2008

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Abstract

We show that from the R^2 high order gravity theory it is possible to produce, in the linearized approch, particles which can be seen like massive modes of gravitational waves (GWs). The presence of the mass generates a longitudinal force in addition of the transverse one which is proper of the massless gravitational waves and the response an interferometer to the effect is computed. This could be, in principle, important to discriminate among the gravity theories. The presence of the mass could also have important applications in cosmology because the fact that gravitational waves can have mass could give a contribution to the dark matter of the Universe.

PACS numbers: 04.80.Nn, 04.30.Nk, 04.50.+h

1 Introduction

The data analysis of interferometric GWs detectors has recently started (for the current status of GWs interferometers see [1, 2, 3, 4, 5, 6, 7, 8]) and the scientific community hopes in a first direct detection of GWs in next years. The results of these detectors will have a fundamental impact on astrophysics and gravitation physics. There will be lots of experimental data to be analyzed, and theorists

will be forced to interact with lots of experiments and data analysts to extract the physics from the data stream.

Detectors for GWs will be important for a better knowledge of the Universe and also to confirm or ruling out the physical consistency of General Relativity or of any other theory of gravitation [9, 10, 11, 12, 13, 14]. This is because, in the context of Extended Theories of Gravity, some differences between General Relativity and the others theories can be pointed out starting by the linearized theory of gravity [9, 10, 12, 14].

In this paper the production and the potential detection with interferometers of a hypotetical massive component of gravitational radiation which arises from the R^2 theory of gravity, which was the first and simplest high order gravity theory proposed [15], is shown.

In the second Section of this paper it is shown that a massive mode of gravitational radiation arises from the high order action [15]

$$S = \int d^4x\sqrt{-g}(R + \alpha R^2) + \mathcal{L}_m. \tag{1}$$

Equation (1) is a particular choice with respect the well known canonical one of general relativity (the Einstein - Hilbert action [16, 17]) which is

$$S = \int d^4x\sqrt{-g}R + \mathcal{L}_m, \tag{2}$$

where R is the Ricci scalar curvature. We empahsize that the presence of the mass could also have important applications in cosmology because the fact that gravitational waves can have mass could give a contribution to the dark matter of the Universe. We also recall that an alternative way to resolve the dark matter and dark energy problems using high order gravity is shown in ref. [18].

In Section three it is shown that the massive component generates a longitudinal force in addition of the transverse one which is proper of the massless case.

After this, in Section four, the potential interferometric detection of this longitudinal component is analyzed and the response of an interferometer is computed. This could be, in principle, important to discriminate among several gravity theories which are today considered.

2 The production of a massive mode of gravitational radiation in the R^2 theory of gravity

If the gravitational Lagrangian is nonlinear in the curvature invariants the Einstein field equations has an order higher than second [9, 12, 13]. For this reason such theories are often called higher-order gravitational theories. This is exactly the case of the action (1).

By varying this action with respect to $g_{\mu\nu}$ (see refs. [12, 13] for a parallel computation) the field equations are obtained (note that in this paper we work with $G=1$, $c=1$ and $\hbar=1$):

$$\begin{array}{c} G_{\mu\nu}=\frac{-4\pi\tilde{G}}{2\alpha R+1}\{+T_{\mu\nu}^{(m)}-\frac{1}{2}g_{\mu\nu}\alpha R^2+ \\ \\ +2\alpha R_{;\mu;\nu}-2\alpha g_{\mu\nu}\Box R\} \end{array} \tag{3}$$

with associed a Klein - Gordon equation for the Ricci curvature scalar

$$\Box R=m^2(R+8\pi\tilde{G}T), \tag{4}$$

where $\Box$ is the d' Alembertian operator and the mass m has been introduced for dimensional motivations: $m^2\equiv-\frac{1}{6\alpha}$, thus α has to be negative [15].

In the above equations $T_{\mu\nu}^{(m)}$ is the ordinary stress-energy tensor of the matter and $\tilde{G}$ is a dimensional, strictly positive, constant [9, 12, 13]. The Newton constant is replaced by the effective coupling

$$G_{eff}=-\frac{1}{2(2\alpha R+1)}, \tag{5}$$

which is different from G. General relativity is obtained when $\alpha=0$.

To study gravitational waves the linearized theory in vacuum ($T_{\mu\nu}^{(m)}=0$) has to be analyzed, with a little perturbation of the background, which is assumed given by the Minkowskian background. In this case the Ricci scalar is assumed slowly varying near zero: $R\simeq 0+\delta R\equiv h_R$.

Putting

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu} \tag{6}$$

to first order in $h_{\mu\nu}$, calling $\widetilde{R}_{\mu\nu\rho\sigma}$, $\widetilde{R}_{\mu\nu}$ and $\widetilde{R}$ the linearized quantity which correspond to $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R, the linearized field equations are obtained [12, 13, 16, 17]:

$$\begin{array}{c} \widetilde{R}_{\mu\nu}-\frac{\widetilde{R}}{2}\eta_{\mu\nu}=\partial_\mu\partial_\nu\widetilde{R}+\eta_{\mu\nu}\Box h_R \\ \\ \Box h_R=m^2h_R. \end{array} \tag{7}$$

$\widetilde{R}_{\mu\nu\rho\sigma}$ and eqs. (7) are invariants for gauge transformations [12, 13]

$$\begin{array}{c} h_{\mu\nu}\rightarrow h'_{\mu\nu}=h_{\mu\nu}-\partial_{(\mu}\epsilon_{\nu)} \\ \\ h_R\rightarrow h'_R=h_R; \end{array} \tag{8}$$

then

$$\bar{h}_{\mu\nu}\equiv h_{\mu\nu}-\frac{h}{2}\eta_{\mu\nu}+\eta_{\mu\nu}h_R \tag{9}$$

can be defined, and, considering the transform for the parameter ϵ^μ

$$\Box\epsilon_\nu = \partial^\mu \bar{h}_{\mu\nu}, \tag{10}$$

a gauge parallel to the Lorenz one of electromagnetic waves can be choosen:

$$\partial^\mu \bar{h}_{\mu\nu} = 0. \tag{11}$$

In this way field equations read like

$$\Box\bar{h}_{\mu\nu} = 0 \tag{12}$$

$$\Box h_R = m^2 h_R \tag{13}$$

Solutions of eqs. (12) and (13) are plan waves:

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p})\exp(ip^\alpha x_\alpha) + c.c. \tag{14}$$

$$h_R = a(\overrightarrow{p})\exp(iq^\alpha x_\alpha) + c.c. \tag{15}$$

where

$$\begin{aligned} k^\alpha &\equiv (\omega, \overrightarrow{p}) \qquad && \omega = p \equiv |\overrightarrow{p}| \\ q^\alpha &\equiv (\omega_m, \overrightarrow{p}) \qquad && \omega_m = \sqrt{m^2 + p^2}. \end{aligned} \tag{16}$$

In eqs. (12) and (14) the equation and the solution for the waves like in standard general relativity [16, 17] have been obtained, but eqs. (13) and (15) are respectively the equation and the solution for the massive mode (see also [12, 13]) arising from the Starobinsky's high order gravity theory.

The fact that the dispersion law for the modes of the massive field h_R is not linear has to be emphatized. The velocity of every tensorial mode $\bar{h}_{\mu\nu}$ is the light speed c, but the dispersion law (the second of eq. (16)) for the modes of h_R is that of a massive field which can be discussed like a wave-packet [12, 13]. Also, the group-velocity of a wave-packet of h_R centered in $\overrightarrow{p}$ is

$$\overrightarrow{v_G} = \frac{\overrightarrow{p}}{\omega}, \tag{17}$$

which is exactly the velocity of a massive particle with mass m and momentum $\overrightarrow{p}$.

From the second of eqs. (16) and eq. (17) it is simple to obtain:

$$v_G = \frac{\sqrt{\omega^2 - m^2}}{\omega}. \tag{18}$$

Then, wanting a constant speed of our wave-packet, it has to be [12, 13]

$$m = \sqrt{(1 - v_G^2)}\omega. \tag{19}$$

The relation (19) is shown in fig. 1 for a value $v_G = 0.9$.

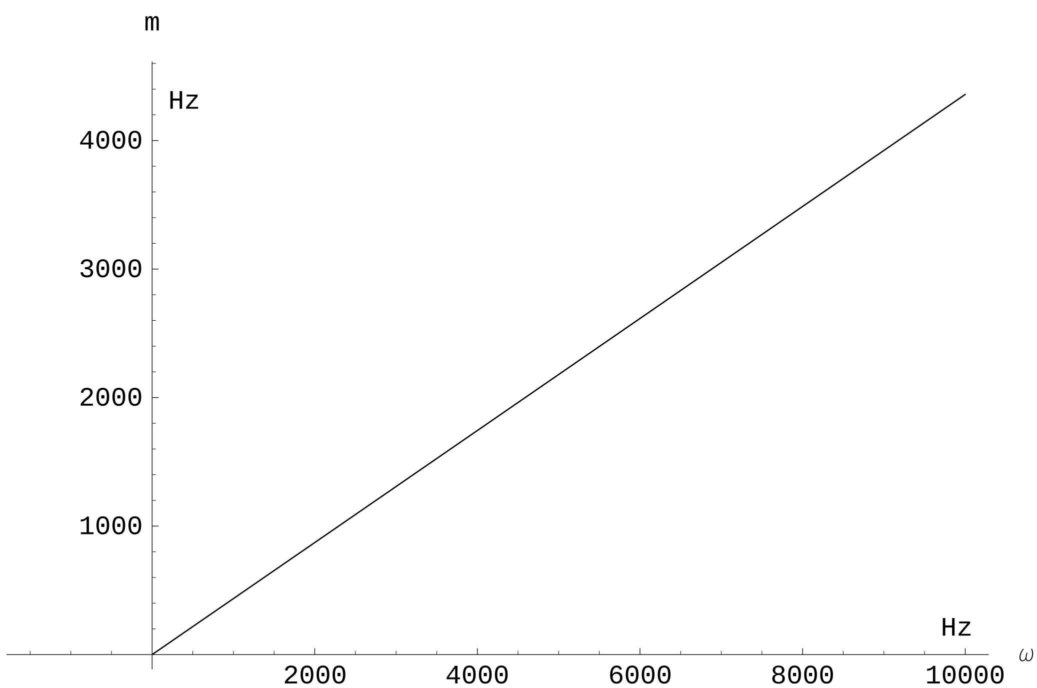

Figure 1: the mass-frequency relation for a massive graviiatonal wave arising from the R^2 high order gravity theory and propagating with a speed of $0.9c$: for the mass it is $1Hz = 10^{-15}eV$

Now the analisys can remain in the Lorenz gauge with trasformations of the type $\Box\epsilon_\nu = 0$; this gauge gives a condition of transversality for the tensorial part of the field: $k^\mu A_{\mu\nu} = 0$, but does not give the transversality for the total field $h_{\mu\nu}$. From eq. (9) it is

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\eta_{\mu\nu} + \eta_{\mu\nu}h_R. \tag{20}$$

At this point, if being in the massless case [17], it could been put

$$\begin{array}{c} \Box\epsilon^\mu = 0 \\ \\ \partial_\mu\epsilon^\mu = -\frac{\bar{h}}{2} + h_R, \end{array} \tag{21}$$

which gives the total transversality of the field. But in the massive case this is impossible. In fact, applying the Dalembertian operator to the second of eqs. (21) and using the field equations (12) and (13) it results

$$\Box\epsilon^\mu = m^2 h_R, \tag{22}$$

which is in contrast with the first of eqs. (21). In the same way it is possible to show that it does not exist any linear relation between the field $\bar{h}_{\mu\nu}$ and h_R. Thus a gauge in wich $h_{\mu\nu}$ is purely spatial cannot be chosen (i.e. it cannot be put $h_{\mu 0} = 0$, see eq. (20)) . But the traceless condition to the field $\bar{h}_{\mu\nu}$ can be put:

$$\begin{array}{c} \Box\epsilon^\mu = 0 \\ \\ \partial_\mu\epsilon^\mu = -\frac{\bar{h}}{2}. \end{array} \tag{23}$$

These equations imply

$$\partial^\mu\bar{h}_{\mu\nu} = 0. \tag{24}$$

To save the conditions $\partial_\mu\bar{h}^{\mu\nu}$ and $\bar{h} = 0$ transformations like

$$\begin{array}{c} \Box\epsilon^\mu = 0 \\ \\ \partial_\mu\epsilon^\mu = 0 \end{array} \tag{25}$$

can be used and, taking $\overrightarrow{p}$ in the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different to zero can be chosen. The condition $\bar{h} = 0$ gives $A_{11} = -A_{22}$. Now, putting these equations in eq. (20) it results

$$h_{\mu\nu}(t,z) = A^+(t-z)e^{(+)}_{\mu\nu} + A^\times(t-z)e^{(\times)}_{\mu\nu} + h_R(t - v_G z)\eta_{\mu\nu}. \tag{26}$$

The term $A^+(t-z)e^{(+)}_{\mu\nu} + A^\times(t-z)e^{(\times)}_{\mu\nu}$ describes the two standard polarizations of gravitational waves which arise from General Relativity, while the term $h_R(t - v_G z)\eta_{\mu\nu}$ is the massive polarization arising from the R^2 theory.

3 The presence of a longitudinal force

The analysis of the two standard polarization is well known in the literature [16, 17]. For a the pure polarization arising by the R^2 theory eq. (26) can be rewritten as

$$h_{\mu\nu}(t - v_G z) = h_R(t - v_G z)\eta_{\mu\nu} \tag{27}$$

and the corrispondent line element is the conformally flat one

$$ds^2 = [1 + h_R(t - v_G z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{28}$$

But, in a laboratory environment on Earth, the coordinate system in which the space-time is locally flat is typically used and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics [12, 13, 16, 17]. This frame is the proper reference frame of a local observer, located for example in the position of the beam splitter of an interferometer. In this frame gravitational waves manifest themself by exerting tidal forces on the masses (the mirror and the beam-splitter in the case of an interferometer). A detailed analysis of the frame of the local observer is given in ref. [17], sect. 13.6. Here only the more important features of this coordinate system are recalled:

the time coordinate x_0 is the proper time of the observer O;

spatial axes are centered in O;

in the special case of zero acceleration and zero rotation the spatial coordinates x_j are the proper distances along the axes and the frame of the local observer reduces to a local Lorentz frame: in this case the line element reads [17]

$$ds^2 = -(dx^0)^2 + \delta_{ij}dx^i dx^j + O(|x^j|^2)dx^\alpha dx^\beta. \tag{29}$$

The effect of the gravitational wave on test masses is described by the equation

$$\ddot{x^i} = -\widetilde{R}^i_{0k0}x^k, \tag{30}$$

which is the equation for geodesic deviation in this frame.

Thus, to study the effect of the massive gravitational wave on test masses, $\widetilde{R}^i_{0k0}$ has to be computed in the proper reference frame of the local observer. But, because the linearized Riemann tensor $\widetilde{R}_{\mu\nu\rho\sigma}$ is invariant under gauge transformations [12, 13, 17], it can be directly computed from eq. (27).

From [17] it is:

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\{\partial_\mu\partial_\beta h_{\alpha\nu} + \partial_\nu\partial_\alpha h_{\mu\beta} - \partial_\alpha\partial_\beta h_{\mu\nu} - \partial_\mu\partial_\nu h_{\alpha\beta}\}, \tag{31}$$

that, in the case eq. (27), begins

$$\widetilde{R}^\alpha_{0\gamma0} = \frac{1}{2}\{\partial^\alpha\partial_0 h_R\eta_{0\gamma} + \partial_0\partial_\gamma h_R\delta^\alpha_0 - \partial^\alpha\partial_\gamma h_R\eta_{00} - \partial_0\partial_0 h_R\delta^\alpha_\gamma\}; \tag{32}$$

the different elements are (only the non zero ones will be written):

$$\partial^{\alpha}\partial_0 h_R \eta_{0\gamma} = \left\{ \begin{array}{ccc} \partial_t^2 h_R & for & \alpha=\gamma=0 \\ \\ -\partial_z\partial_t h_R & for & \alpha=3;\gamma=0 \end{array} \right\} \tag{33}$$

$$\partial_0\partial_{\gamma} h_R \delta_0^{\alpha} = \left\{ \begin{array}{ccc} \partial_t^2 h_R & for & \alpha=\gamma=0 \\ \\ \partial_t\partial_z h_R & for & \alpha=0;\gamma=3 \end{array} \right\} \tag{34}$$

$$-\partial^{\alpha}\partial_{\gamma} h_R \eta_{00} = \partial^{\alpha}\partial_{\gamma} h_R = \left\{ \begin{array}{ccc} -\partial_t^2 h_R & for & \alpha=\gamma=0 \\ \\ \partial_z^2 h_R & for & \alpha=\gamma=3 \\ \\ -\partial_t\partial_z h_R & for & \alpha=0;\gamma=3 \\ \\ \partial_z\partial_t h_R & for & \alpha=3;\gamma=0 \end{array} \right\} \tag{35}$$

$$-\partial_0\partial_0 h_R \delta_{\gamma}^{\alpha} = \;\; -\partial_z^2 h_R \quad for \quad \alpha=\gamma \;\; . \tag{36}$$

Now, putting these results in eq. (32) it results:

$$\begin{array}{c} \widetilde{R}^1_{010} = -\frac{1}{2}\ddot{h}_R \\ \\ \widetilde{R}^2_{010} = -\frac{1}{2}\ddot{h}_R \\ \\ \widetilde{R}^3_{030} = \frac{1}{2}\Box h_R. \end{array} \tag{37}$$

But, putting the field equation (13) in the third of eqs. (37) it is

$$\widetilde{R}^3_{030} = \frac{1}{2}m^2 h_R, \tag{38}$$

which shows that the field is not transversal.
Infact, using eq. (30) it results

$$\ddot{x} = \frac{1}{2}\ddot{h}_R x, \tag{39}$$

$$\ddot{y} = \frac{1}{2}\ddot{h}_R y \tag{40}$$

and

$$\ddot{z} = -\frac{1}{2}m^2 h_R(t - v_G z)z. \tag{41}$$

Then the effect of the mass is the generation of a *longitudinal* force (in addition to the transverse one). Note that in the limit $m \to 0$ the longitudinal force vanishes.

4 The interferometer's response to the longitudinal component

Before starting the analysis it has to be discussed if there are fenomenogical limitations to the mass of the wave [12, 13]. Treating h_R like a classical wave, that acts coherently with the interferometer, it has to be $m \ll 1/L$, where $L = 3$ kilometers in the case of Virgo and $L = 4$ kilometers in the case of LIGO. Thus it has to be approximately $m < 10^{-9}eV$. However there is a stronger limitation coming from the fact that the massive wave needs a frequency which falls in the frequency-range for earth based gravitational antennas that is the interval $10Hz \leq f \leq 10KHz$ [1, 2, 3, 4, 5, 6, 7, 8]. For a massive gravitational wave, from the second of eqs. (16) it is:

$$2\pi f = \omega = \sqrt{m^2 + p^2}, \tag{42}$$

were p is the momentum [13]. Thus it needs

$$0eV \leq m \leq 10^{-11}eV. \tag{43}$$

For these light particles their effect can be still discussed as a coherent gravitational wave. For the discussion of this longitudinal effect we start directly from the gauge (28).

Eq. (28) can be rewritten as

$$(\frac{dt}{d\tau})^2 - (\frac{dx}{d\tau})^2 - (\frac{dy}{d\tau})^2 - (\frac{dz}{d\tau})^2 = \frac{1}{(1 + h_R)}, \tag{44}$$

where τ is the proper time of the test masses.

From eqs. (28) and (44) the geodesic equations of motion for test masses (i.e. the beam-splitter and the mirrors of the interferometer), can be obtained

$$\begin{array}{ccc}
\frac{d^2x}{d\tau^2} & = & 0 \\
\\
\frac{d^2y}{d\tau^2} & = & 0 \\
\\
\frac{d^2t}{d\tau^2} & = & \frac{1}{2}\frac{\partial_t(1+h_R)}{(1+h_R)^2} \\
\\
\frac{d^2z}{d\tau^2} & = & -\frac{1}{2}\frac{\partial_z(1+h_R)}{(1+h_R)^2}.
\end{array} \tag{45}$$

The first and the second of eqs. (45) can be immediately integrated obtaining

$$\frac{dx}{d\tau} = C_1 = const. \tag{46}$$

$$\frac{dy}{d\tau} = C_2 = const. \tag{47}$$

In this way eq. (44) becomes

$$(\frac{dt}{d\tau})^2-(\frac{dz}{d\tau})^2=\frac{1}{(1+h_R)}. \tag{48}$$

If we assume that test masses are at rest initially we get $C_1=C_2=0$. Thus we see that, even if the GW arrives at test masses, we do not have motion of test masses within the $x-y$ plane in this gauge. We could understand this directly from eq. (28) because the absence of the x and of the y dependences in the metric implies that test masses momentum in these directions (i.e. C_1 and C_2 respectively) is conserved. This results, for example, from the fact that in this case the x and y coordinates do not esplicitly enter in the Hamilton-Jacobi equation for a test mass in a gravitational field [16].

Now we will see that, in presence of the GW, we have motion of test masses in the z direction which is the direction of the propagating wave. An analysis of eqs. (45) shows that, to simplify equations, we can introduce the retarded and advanced time coordinates (u,v):

$$\begin{aligned} u&=t-v_G z \\ v&=t+v_G z. \end{aligned} \tag{49}$$

From the third and the fourth of eqs. (45) we have

$$\frac{d}{d\tau}\frac{du}{d\tau}=\frac{\partial_v[1+h_R(u)]}{(1+h_R(u))^2}=0. \tag{50}$$

This equation can be integrated obtaining

$$\frac{du}{d\tau}=\alpha, \tag{51}$$

where α is an integration constant. From eqs. (48) and (51), we also get

$$\frac{dv}{d\tau}=\frac{\beta}{1+h_R} \tag{52}$$

where $\beta\equiv\frac{1}{\alpha}$, and

$$\tau=\beta u+\gamma, \tag{53}$$

where the integration constant γ correspondes simply to the retarded time coordinate translation u. Thus, without loss of generality, we can put it equal to zero. Now let us see what is the meaning of the other integration constant β. We can write the equation for z from eqs. (51) and (52):

$$\frac{dz}{d\tau}=\frac{1}{2\beta}(\frac{\beta^2}{1+h_R}-1). \tag{54}$$

When it is $h_R = 0$ (i.e. before the GW arrives at the test masses) eq. (54) becomes

$$\frac{dz}{d\tau} = \frac{1}{2\beta}(\beta^2 - 1). \tag{55}$$

But this is exactly the initial velocity of the test mass, so we have to choose $\beta = 1$ because we suppose that test masses are at rest initially. This also imply $\alpha = 1$.

To find the motion of a test mass in the z direction we see that from eq. (53) we have $d\tau = du$, while from eq. (52) we have $dv = \frac{d\tau}{1+h_R}$. Because it is $v_G z = \frac{v-u}{2}$ we obtain

$$dz = \frac{1}{2v_G}(\frac{d\tau}{1+h_R} - du), \tag{56}$$

which can be integrated as

$$\begin{array}{c} z = z_0 + \frac{1}{2v_G}\int(\frac{du}{1+h_R} - du) = \\ \\ = z_0 - \frac{1}{2v_G}\int_{-\infty}^{t-v_G z}\frac{h_R(u)}{1+h_R(u)}du, \end{array} \tag{57}$$

where z_0 is the initial position of the test mass. Now the displacement of the test mass in the z direction can be written as

$$\begin{array}{c} \Delta z = z - z_0 = -\frac{1}{2v_G}\int_{-\infty}^{t-v_G z_0 - v_G \Delta z}\frac{h_R(u)}{1+h_R(u)}du \\ \\ \simeq -\frac{1}{2v_G}\int_{-\infty}^{t-v_G z_0}\frac{h_R(u)}{1+h_R(u)}du. \end{array} \tag{58}$$

We can also rewrite our results in function of the time coordinate t:

$$\begin{array}{ccc} x(t) & = & x_0 \\ \\ y(t) & = & y_0 \\ \\ z(t) & = & z_0 - \frac{1}{2v_G}\int_{-\infty}^{t-v_G z_0}\frac{h_R(u)}{1+h_R(u)}d(u) \\ \\ \tau(t) & = & t - v_G z(t), \end{array} \tag{59}$$

Calling l and $L+l$ the unperturbed positions of the beam-splitter and of the mirror and using the third of eqs. (59) the varying position of the beam-splitter and of the mirror are given by

$$\begin{array}{c} z_{BS}(t) = l - \frac{1}{2v_G}\int_{-\infty}^{t-v_G l}\frac{h_R(u)}{1+h_R(u)}d(u) \\ \\ z_M(t) = L + l - \frac{1}{2v_G}\int_{-\infty}^{t-v_G (L+l)}\frac{h_R(u)}{1+h_R(u)}d(u) \end{array} \tag{60}$$

But we are interested in variations in the proper distance (time) of test masses, thus, in correspondence of eqs. (60), using the fourth of eqs. (59) we

get

$$\tau_{BS}(t) = t - v_G l - \tfrac{1}{2}\int_{-\infty}^{t-v_G l} \tfrac{h_R(u)}{1+h_R(u)} d(u)$$
$$\tau_M(t) = t - v_G L - v_G l - \tfrac{1}{2}\int_{-\infty}^{t-v_G(L+l)} \tfrac{h_R(u)}{1+h_R(u)} d(u). \tag{61}$$

Then the total variation of the proper time is given by

$$\triangle\,\tau(t) = \tau_M(t) - \tau_{BS}(t) = v_G L - \frac{1}{2}\int_{t-v_G l}^{t-v_G(L+l)} \frac{h_R(u)}{1+h_R(u)} d(u). \tag{62}$$

In this way, recalling that in the used units the unperturbed proper distance (time)is $T = L$, the difference between the total variation of the proper time in presence and the total variation of the proper time in absence of the GW is

$$\delta\tau(t) \equiv \triangle\tau(t) - L = -L(v_G + 1) - \frac{1}{2}\int_{t-v_G l}^{t-v_G(L+l)} \frac{h_R(u)}{1+h_R(u)} d(u). \tag{63}$$

This quantity can be computed in the frequency domain, defining the Fourier transform of h_R as

$$\widetilde{h}_R(\omega) = \int_{-\infty}^{\infty} dt\ h_R(t)\exp(i\omega t). \tag{64}$$

and using the translation and derivation Fourier theorems, obtaining

$$\delta\widetilde{\tau}(\omega) = L(1 - v_G^2)\exp[i\omega L(1+v_G)] + \tfrac{L}{2\omega L(v_G^2-1)^2}$$
$$[\exp[2i\omega L](v_G+1)^3(-2i + \omega L(v_G-1) + 2L\exp[i\omega L(1+v_G)] \tag{65}$$
$$(6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4) + L(v_G+1)^3(-2i + \omega L(v_G+1))]\widetilde{h}_R.$$

A “signal” can be also defined:

$$\widetilde{S}(\omega) \equiv \tfrac{\delta\widetilde{\tau}(\omega)}{L} = (1 - v_G^2)\exp[i\omega L(1+v_G)] + \tfrac{1}{2\omega L(v_G^2-1)^2}$$
$$[\exp[2i\omega L](v_G+1)^3(-2i + \omega L(v_G-1) + 2\exp[i\omega L(1+v_G)] \tag{66}$$
$$(6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4) + (v_G+1)^3(-2i + \omega L(v_G+1))]\widetilde{h}_R.$$

Then the function

$$\Upsilon_l(\omega) \equiv (1 - v_G^2)\exp[i\omega L(1+v_G)] + \tfrac{1}{2\omega L(v_G^2-1)^2}$$
$$[\exp[2i\omega L](v_G+1)^3(-2i + \omega L(v_G-1) + 2\exp[i\omega L(1+v_G)] \tag{67}$$
$$(6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4) + (v_G+1)^3(-2i + \omega L(v_G+1))],$$

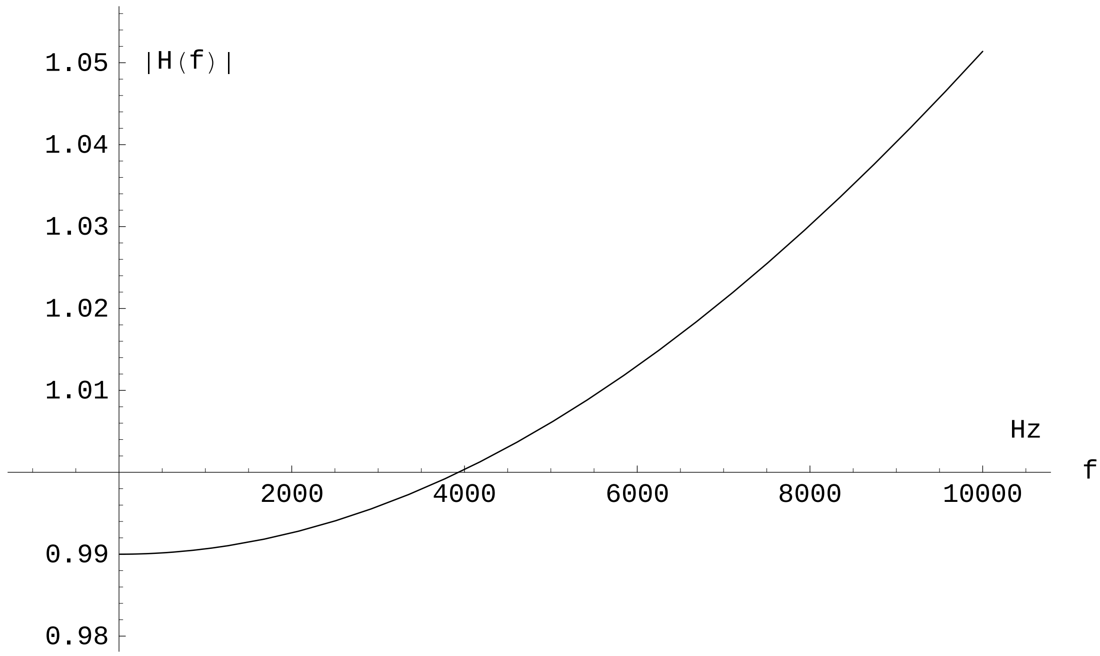

Figure 2: the absolute value of the longitudinal response function (65) of the Virgo interferometer ($L = 3Km$) to a GW arising from the R^2 high order gravity theory and propagating with a speed of $0.1c$ (non relativistic case).

is the response function of an arm of our interferometer located in the z-axis, due to the longitudinal component of the massive gravitational wave arising from the R^2 high order gravity theory and propagating in the same direction of the axis.

For $v_G \to 1$ it is $\Upsilon_l(\omega) \to 0$.

In figures 2, 3 and 4 are shown the response functions (67) for an arm of the Virgo interferometer ($L = 3Km$) for $v_G = 0.1$ (non-relativistic case), $v_G = 0.9$ (relativistic case) and $v_G = 0.999$ (ultra-relativistic case). We see that in the non-relativistic case the signal is stronger as it could be expected (for $m \to 0$ we expect$\Upsilon_l(\omega) \to 0$). In figures 5, 6, and 7 the same response functions are shown for the Ligo interferometer ($L = 4Km$).

5 Conclusions

We have shown that from the R^2 high order gravity theory it is possible to produce, in the linearized approch, particles which can be seen like massive modes of gravitational waves. The presence of the mass generates a longitudinal force in addition of the transverse one which is proper of the massless gravitational

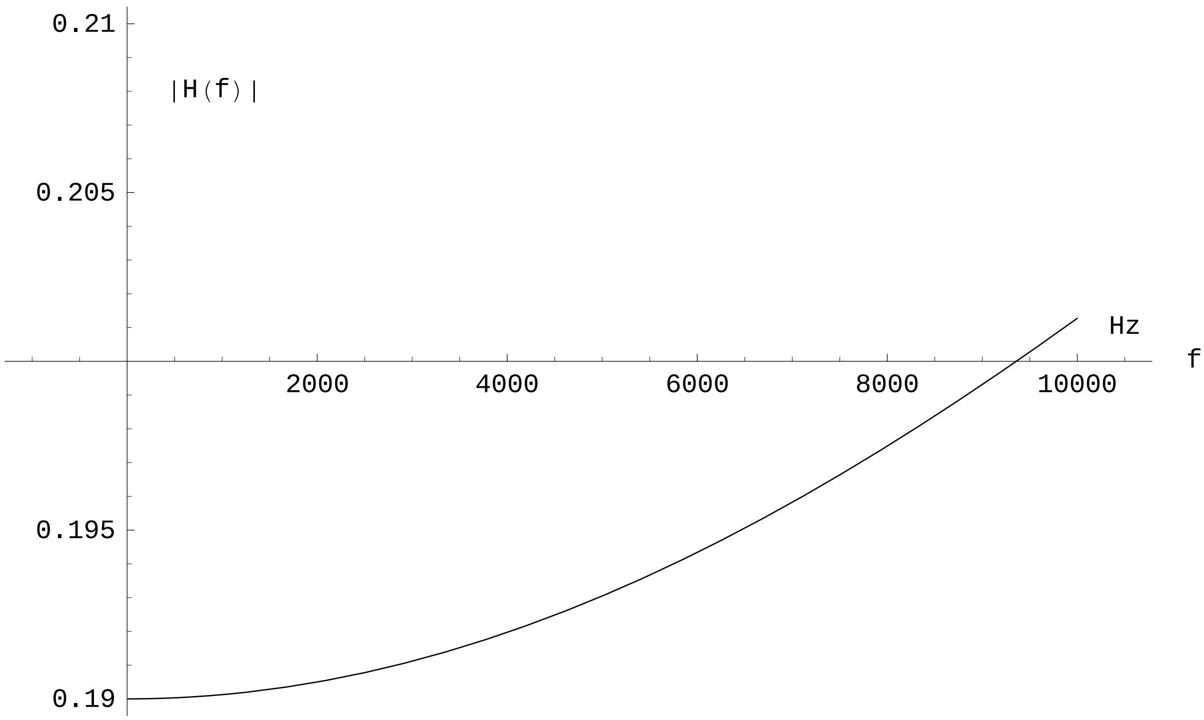

Figure 3: the absolute value of the longitudinal response function (65) of the Virgo interferometer ($L = 3Km$) to a GW arising from the R^2 high order gravity theory and propagating with a speed of 0.9 (relativistic case).

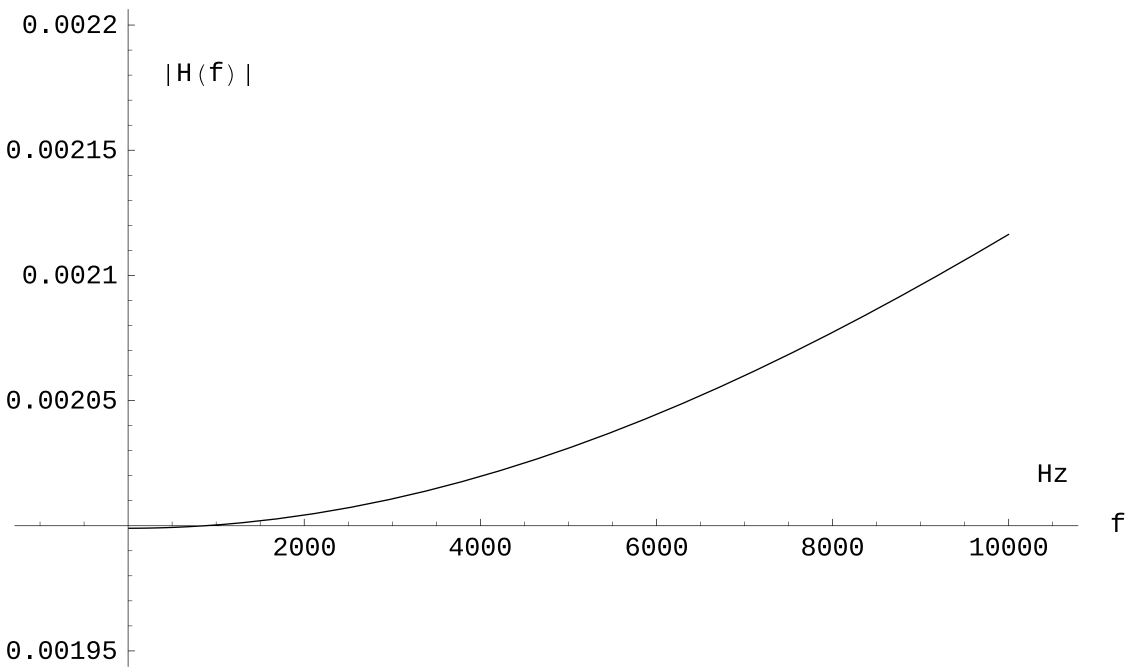

Figure 4: the absolute value of the longitudinal response function (65) of the Virgo interferometer ($L = 3Km$) to a GW arising from the R^2 high order gravity theory and propagating with a speed of 0.999 (ultra relativistic case).

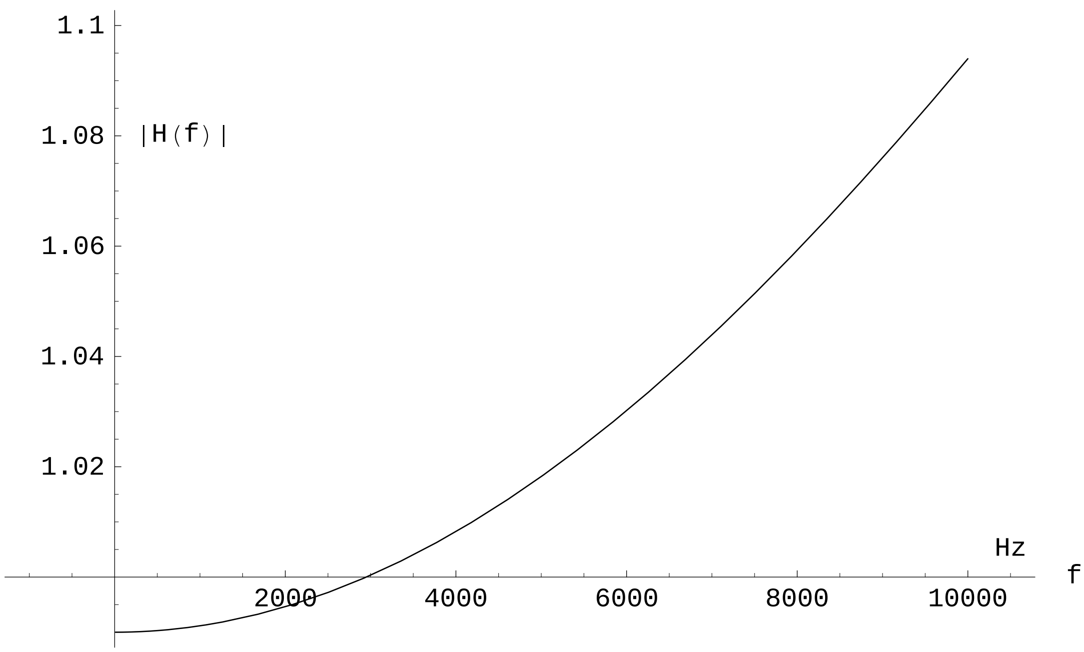

Figure 5: the absolute value of the longitudinal response function (65) of the LIGO interferometer ($L = 4Km$) to a GW arising from the R^2 high order gravity theory and propagating with a speed of $0.1c$ (non relativistic case).

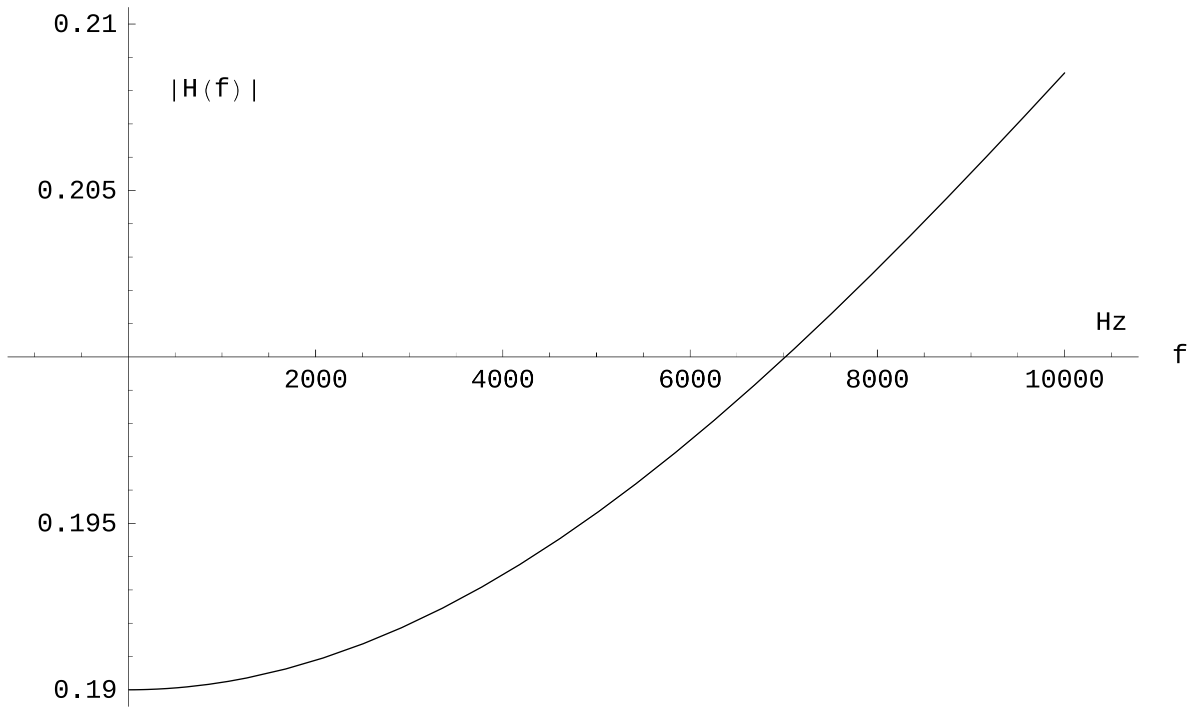

Figure 6: the absolute value of the longitudinal response function (65) of the LIGO interferometer ($L = 4Km$) to a GW arising from the R^2 high order gravity theory and propagating with a speed of $0.9c$ (relativistic case).

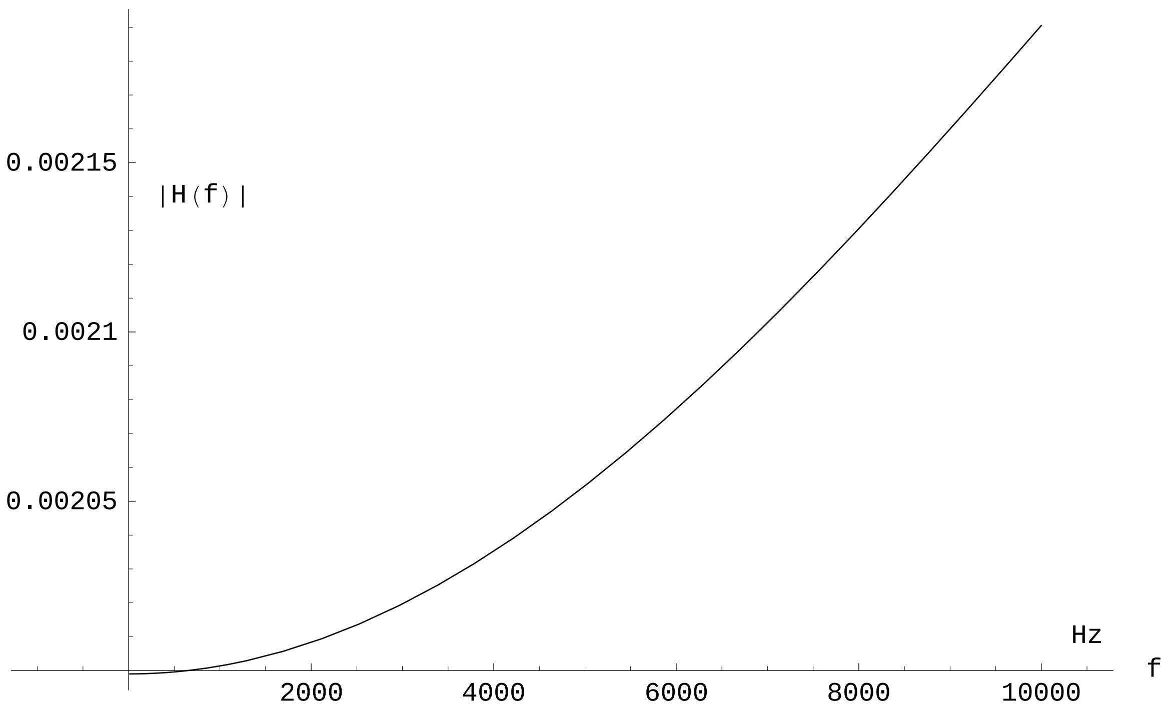

Figure 7: the absolute value of the longitudinal response function (65) of the LIGO interferometer ($L = 4Km$) to a GW arising from the R^2 high order gravity theory and propagating with a speed of $0.999c$ (ultra relativistic case).

waves and the response an interferometer to the effect has been computed. The presence of the mass could also have important applications in cosmology because the fact that gravitational waves can have mass could give a contribution to the dark matter of the Universe. As a final remark, we recall that the potential detection of a longitudinal component of GWs could be, in principle, an useful tool to discriminate among several gravity theories which are today considered.

6 Acknowledgements

I would like to thank Salvatore Capozziello, Maria Felicia De Laurentis and Franceso Rubanu for helpful advices during our work. I have to thank the European Gravitational Observatory (EGO) consortium for the using of computing facilities.

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